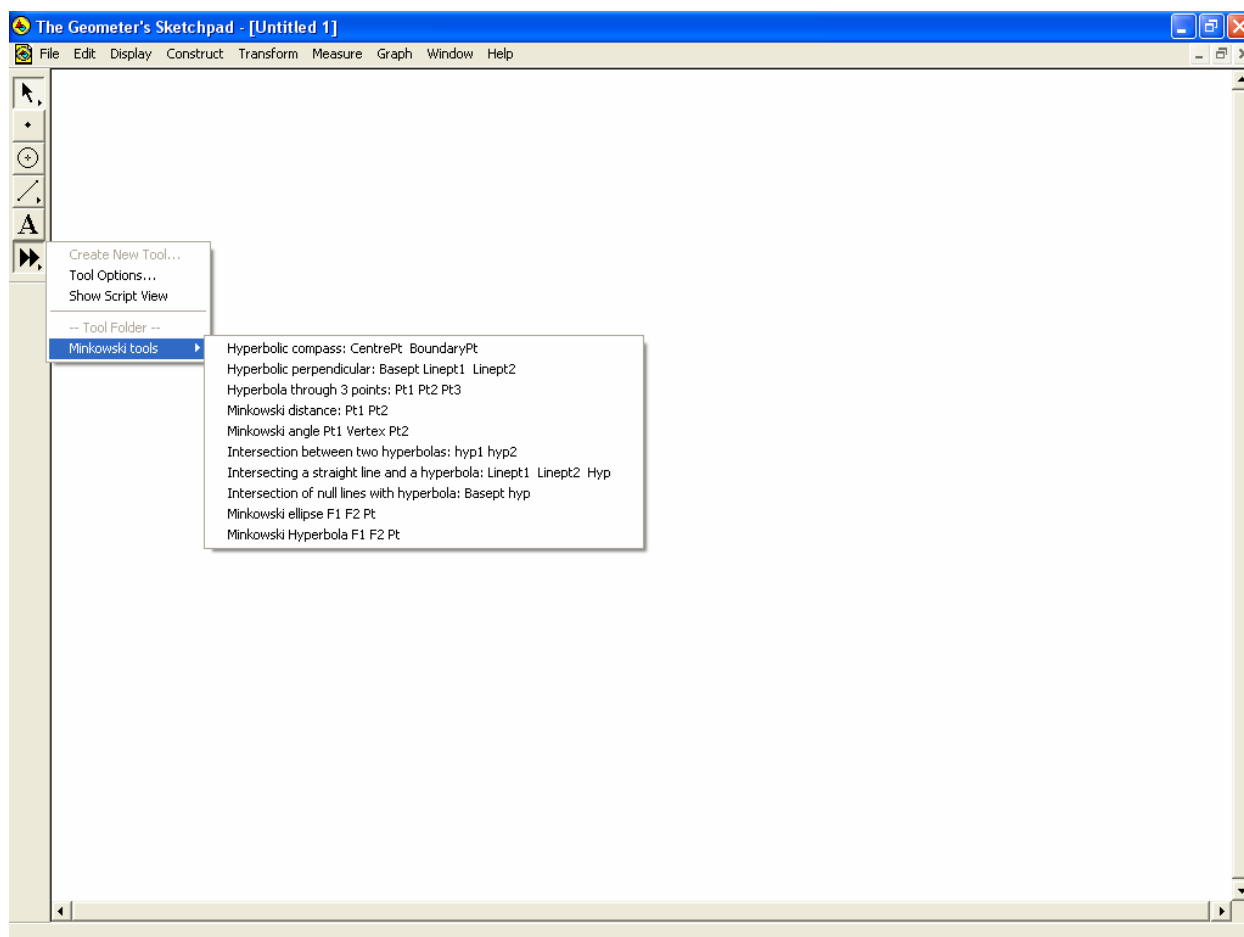


# Minkowski Tools for SketchPad

The purpose of the Minkowski tools is to provide the user with a set of tools that exchange those Euclidean tools that are no longer valid. They are stored in a SketchPad document

## Minkowski tools.gsp

and once this is loaded you get access to the additional tools on the Macro tool while you are working on other documents. But you can also add it permanently to the folder **Tool Folder** within the **SketchPad folder**. It will then be added to the macro tools automatically every time the program opens:

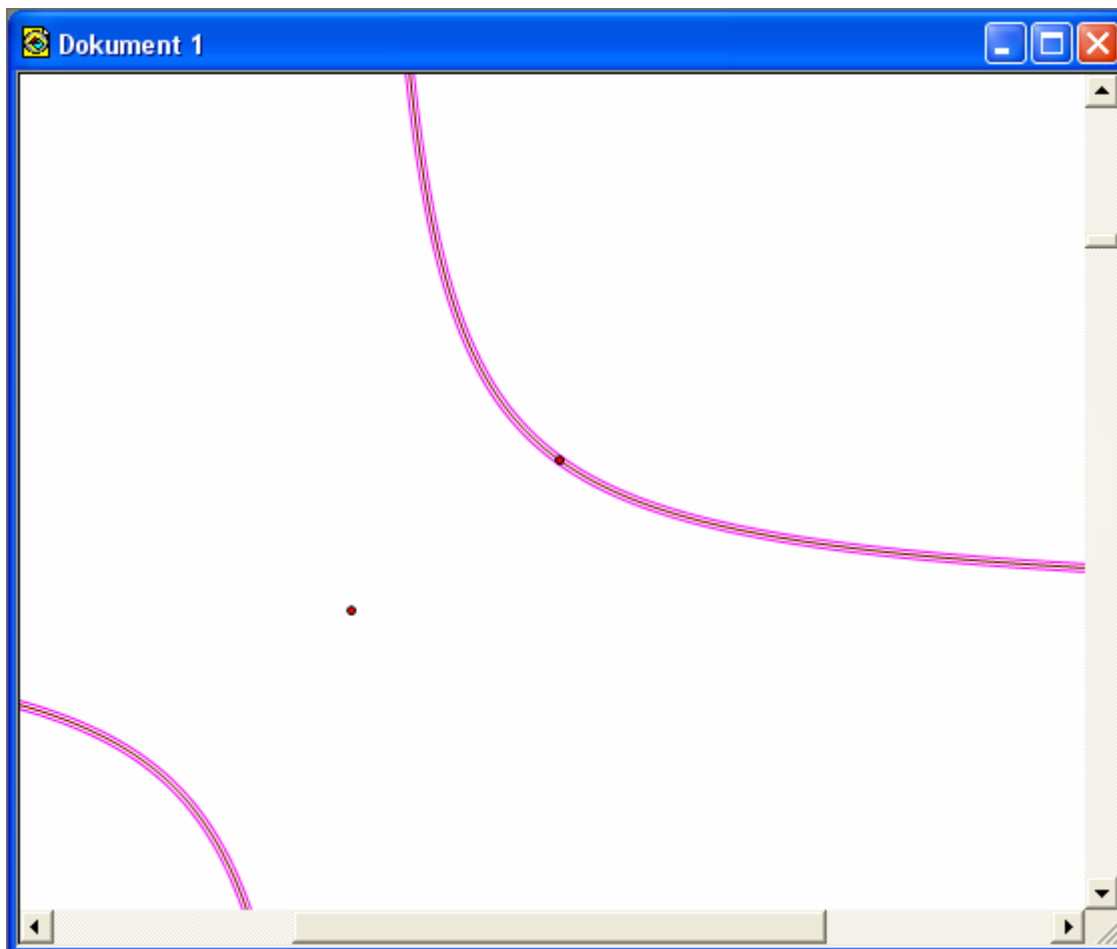


The basic **construction tools** are the Hyperbolic compass and the Hyperbolic perpendicular. They are supplemented with a tool that constructs the hyperbola through 3 points. The basic **measurement tools** are the Minkowski distance and the Minkowski angle (which is only defined for proper angles). Next comes some special tools for constructing **intersection points** with the hyperbolas. Finally there are some general tools for constructing the central **conic sections** the ellipse and the hyperbola using the Minkowski focal points.

All other tools are either the same as in the Euclidean Geometry (e.g. parallel lines, midpoints, translations, point reflections etc.) or they must be constructed on the fly by the user. Notice that we still have Euclidean tools available that fail in Minkowski Geometry. So you can still find e.g. perpendiculars or rotations, but these should only be used for the Euclidean investigations, not the Minkowski investigations.

## Hyperbolic compass: CentrePt BoundaryPt

This is the basic tool drawing an equiangular hyperbola with horizontal and vertical axes given a centre point and a boundary point. To use this tool you therefore select two points: The first being the centre, the second being the boundary point. Once the first point has been selected the equiangular hyperbola with horizontal and vertical axes pops up and follows the second point around on the screen until this is finally fixed:



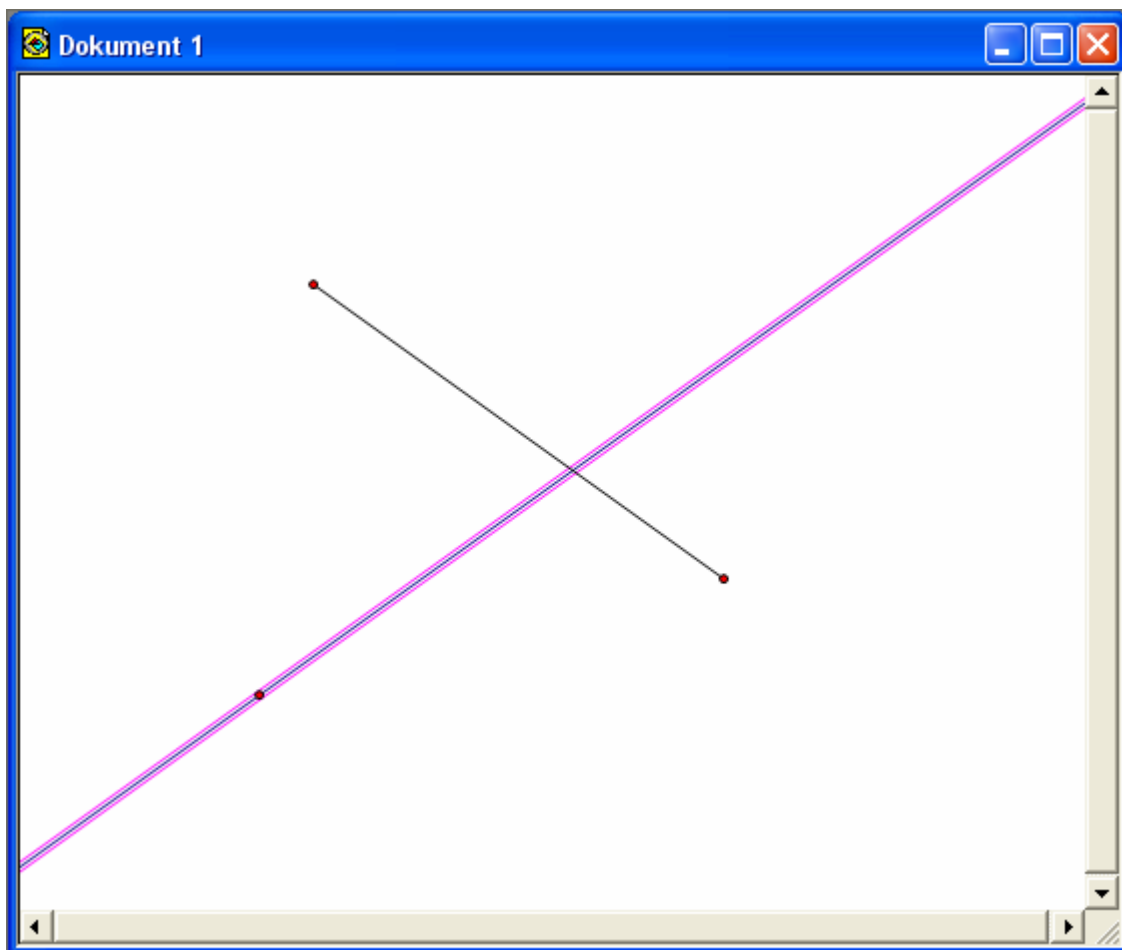
The hyperbola is constructed as a locus. As a consequence you can not ask for its equation and you can not construct intersection points automatically as is possible for the circle. For this reasons special tools are included that allow construction of intersection points when a Minkowski hyperbola is involved.

It is presupposed that the two points do not fall on a null line, i.e. a horizontal or vertical line.

*Remark:* The asymptotes of the hyperbola are not included. If you need these the most robust way of drawing horizontal and vertical lines is in fact to translate the centre point in a horizontal and vertical direction. You can also do it simply drawing a line holding **SHIFT** down while you are drawing, but then the directions will not be stable if you incorporate the construction in a macro.

## Hyperbolic perpendicular: Basept Lpt1 Lpt2

This is the basic tool drawing perpendiculars in Minkowski Geometry. It results in a line passing through a given point – the base point – perpendicular to a given straight object, i.e. a line, a ray, a segment or a vector. To maintain compatibility across platforms the straight object is specified through two points, line point 1 and line point 2, thus avoiding handling the various cases separately. To use this tool you must therefore select three points, the first one being the base point and the remaining two specifying the direction of the straight object. Once the base point and the first line point has been selected a line being perpendicular to the straight object pops up and follows the second line appoint around on the screen until its position is finally fixed:

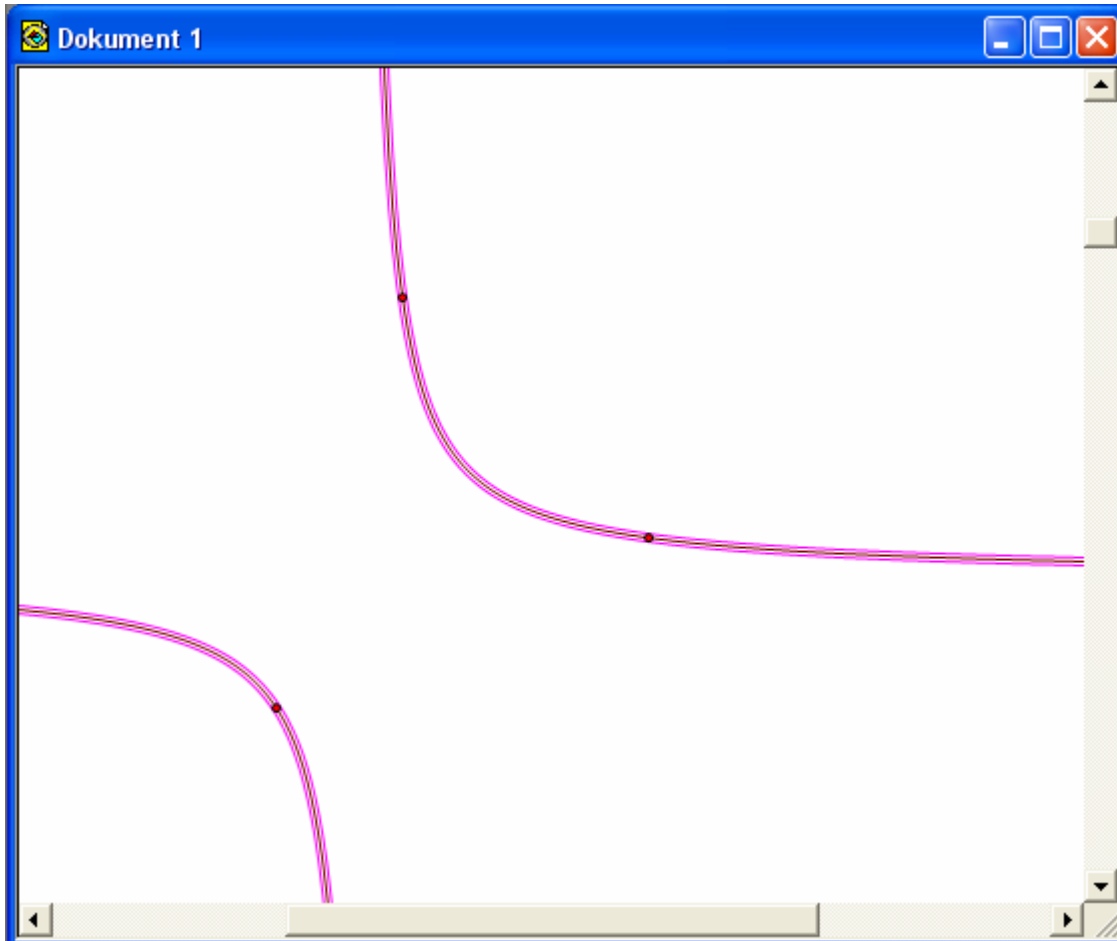


The perpendicular in Minkowski Geometry is the line with the opposite slope of the given straight object. If you need to check if two straight lines are perpendicular in the Minkowski sense, you can therefore check if the ratio of their slopes is  $-1$  (where as it is the product of the slopes, that has to be  $-1$  in Euclidean Geometry).

Notice that a perpendicular bisector is not included as a basic tool. It can be constructed on the fly by first constructing the midpoint and then the perpendicular through the midpoint. This will work, because the midpoints are common to both Euclidean and Minkowski Geometry.

## Hyperbola through 3 points: Pt1 Pt2 Pt3

This is a supplementary tool that allows you to draw an equiangular hyperbola passing through three points directly. To use this tool you must therefore select three points. Once the two first points have been selected the equiangular hyperbola pops up and follows the third point around on the screen until its position is finally fixed:



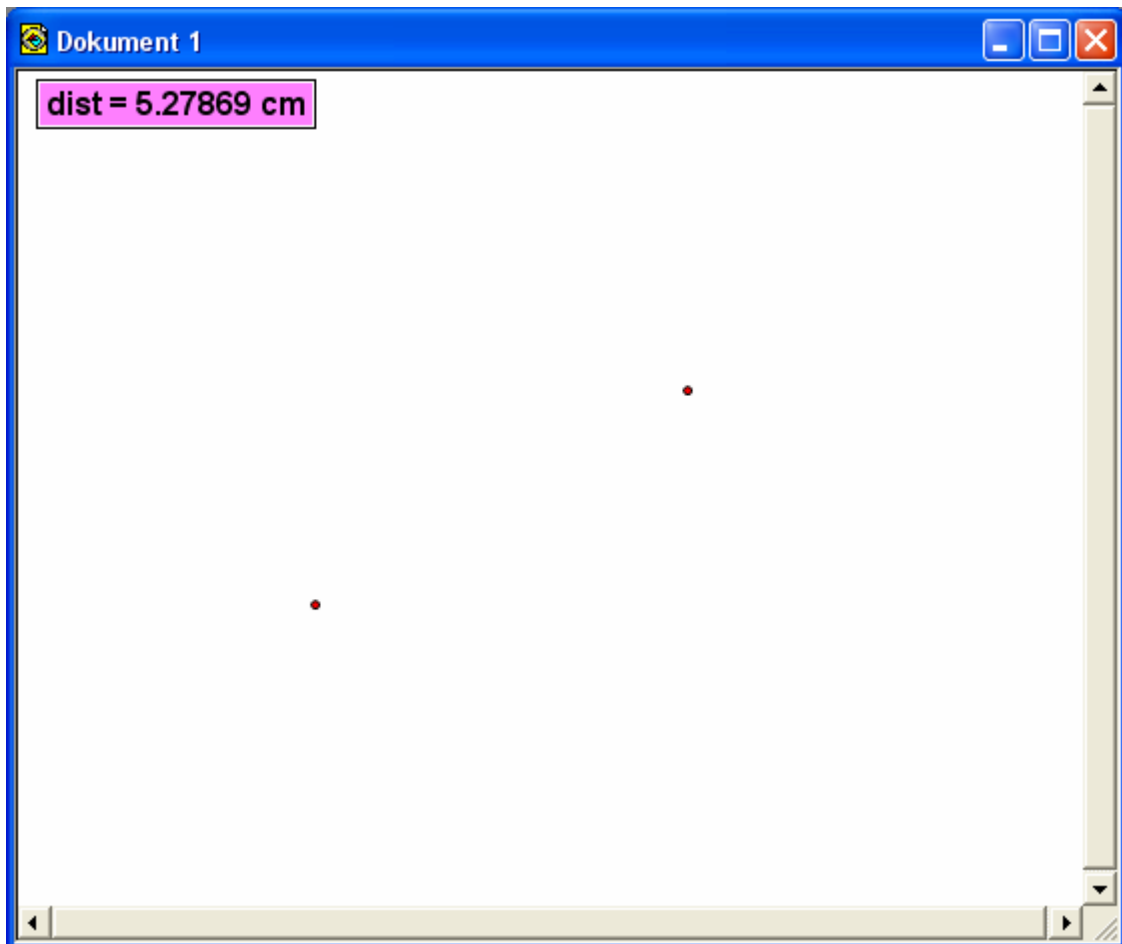
It is of course equivalent to the construction of the circumscribed hyperbola for a triangle and as such it could have been constructed on the fly using the common intersection point of the three perpendicular bisectors as a centre point. But in many more complex constructions it is cumbersome to repeat this sub construction again and again. E.g. it will allow you to construct the nine-point hyperbola associated with a triangle directly as the hyperbola passing through the three midpoints.

As in the Euclidean case it is presupposed that the three points do not lie on a straight line. In that case the hyperbola degenerates to a straight line passing through the three points.

Furthermore it is presupposed that no two pair of points fall on a null line, i.e. they must not lie on the same horizontal or vertical line. In that case the hyperbola degenerates to a pair of null lines passing through the three points.

## Minkowski distance: Pt1 Pt2

This is the basic tool for measuring distances between two points. To use this tool you must therefore select two points. Once the first point has been selected, the resulting distance pop up and follows the second point around until its position is finally fixed:



Notice that the distance is always measured in cm whether or not the segment is time like or space like (positive slope or negative slope). In special relativity it is usually the other way around: There all distances in space time are measured in seconds (with spatial distances often actually measured in light seconds, but that is the same in conventional units, where the speed of light is 1). There are also special treatments of special relativity, where space-like distances are real, whereas time-like distances are imaginary. But we will ignore all such attempts of distinguishing between space like and time like distances.

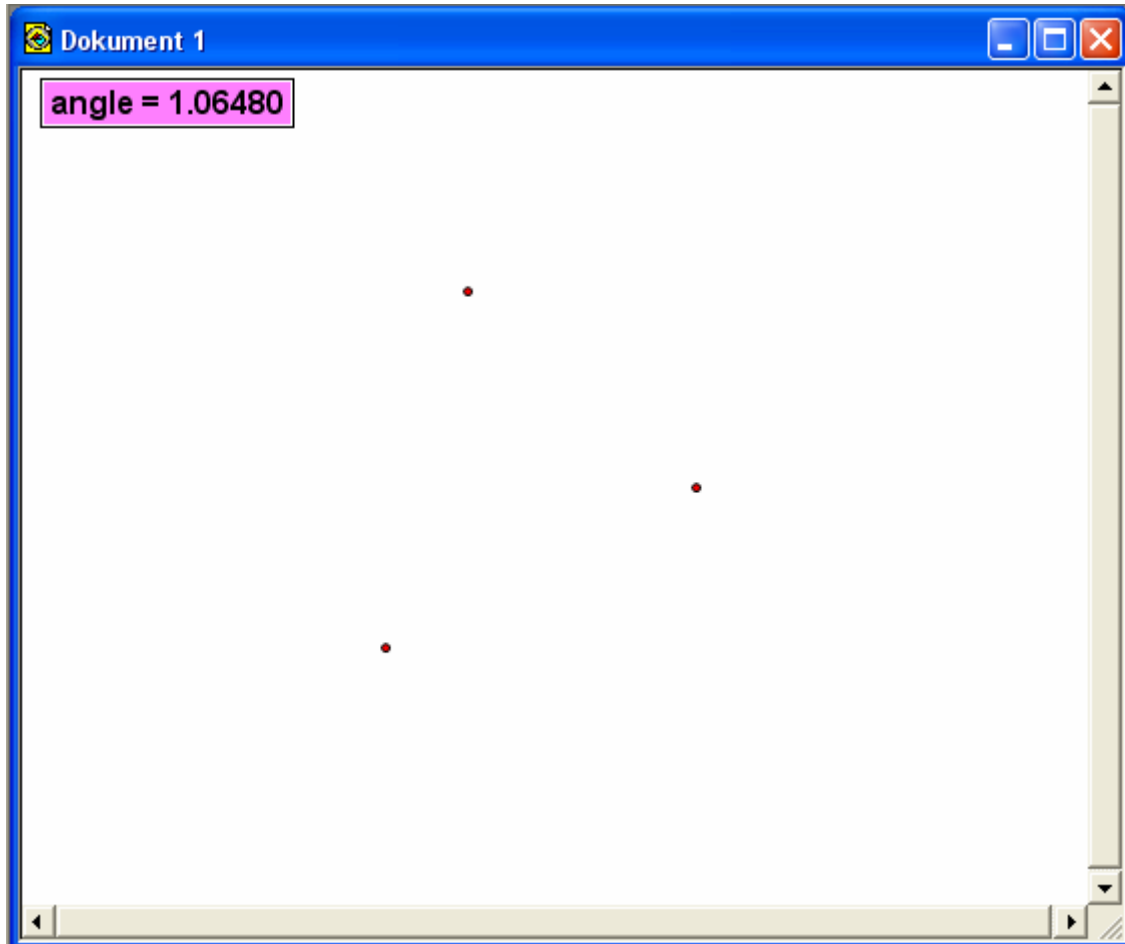
In terms of conventional rectangular coordinates the Minkowski distance is computed using the formula:

$$dist = \sqrt{2 \cdot |x_2 - x_1| \cdot |y_2 - y_1|}$$

Especially horizontal and vertical distances are always zero.

## Minkowski angle: Pt1 Vertex Pt2

This is the basic tool for measuring angles. To use this tool the angle must be specified through three points, with the vertex being the middle point. Therefore you must select three points. Once the first two points have been selected, the resulting angle pop up and follows the third point around until its position is fixed:



The angle is measured in radians (i.e. the arc length of the corresponding unit hyperbolic arc). Notice that only proper angles (with the ratio of their slopes being positive) are measured. For improper angles the measurement results in the answer: non-existent. Notice also that angles are oriented in the usual way: Counterclockwise angles are positive.

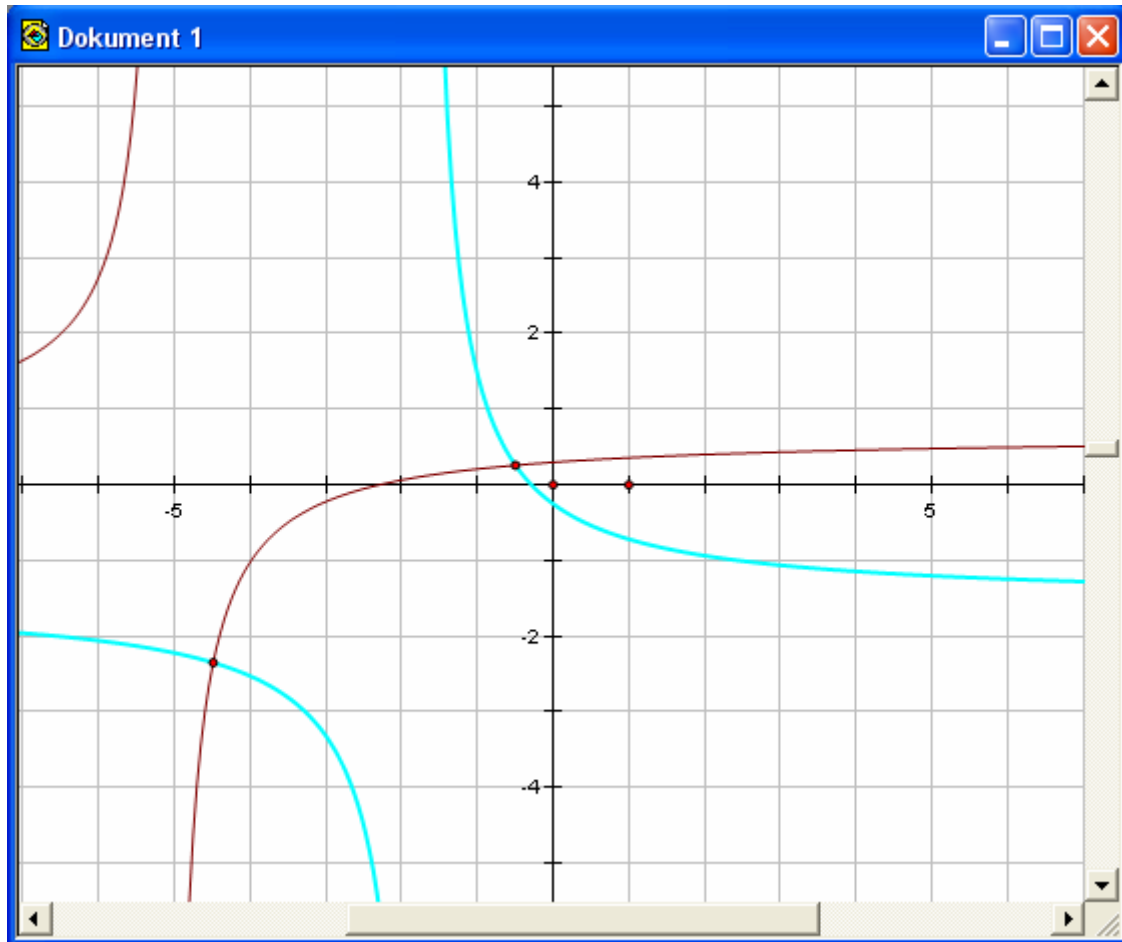
There are also special treatments of special relativity, where proper angles are real, whereas improper angles are imaginary (with a right angle having the radian measure  $i\pi/2$ ). But we will ignore all such attempts of distinguishing between proper and improper angles.

In terms of conventional slopes the Minkowski angle is computed using the formula:

$$angle = \frac{1}{2} \cdot \ln \left( \frac{slope_2}{slope_1} \right).$$

## Intersection between two hyperbolas: Hyp1 Hyp2

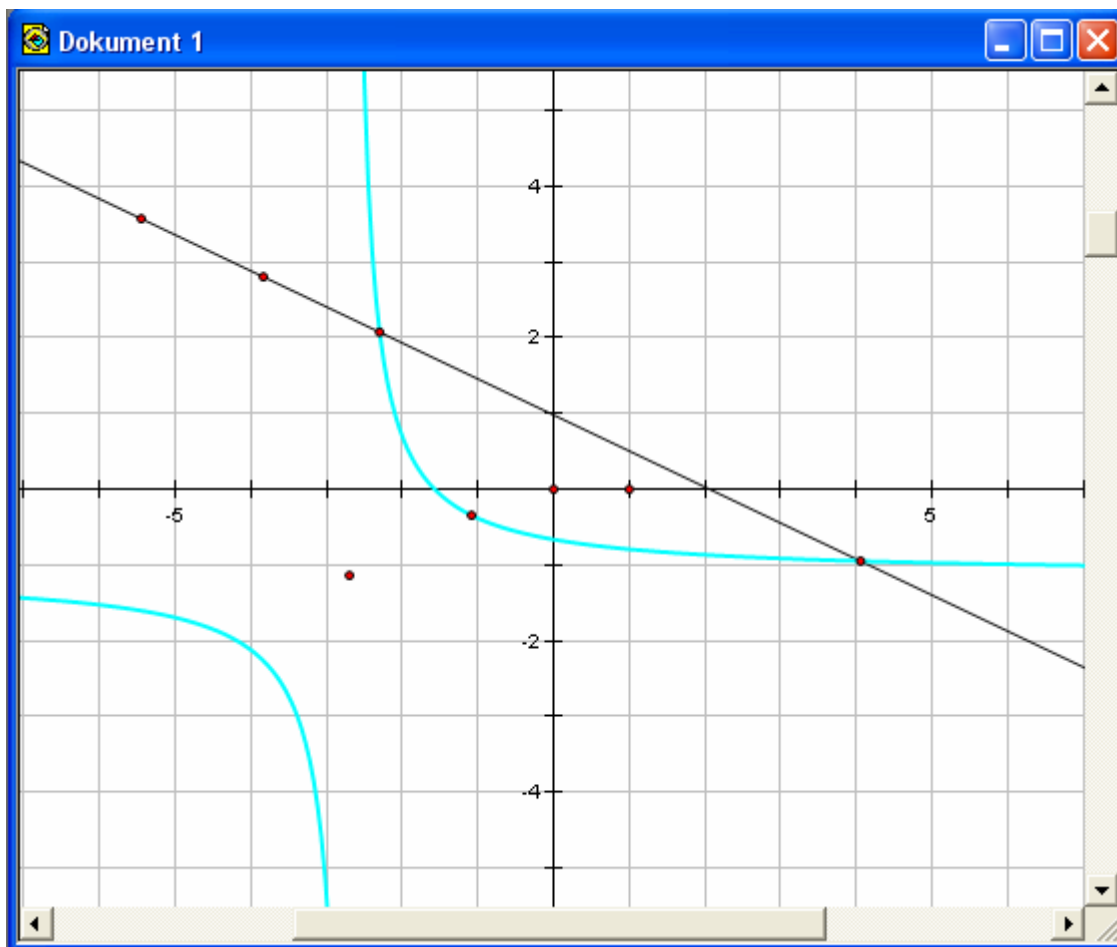
This is the basic tool for constructing an intersecting point between two hyperbolas drawn using the hyperbolic compass. The position of the intersection points are computed using an underlying coordinate system, so when this macro is invoked a coordinate system automatically pops up. You can hide the coordinate system if it disturbs your investigations. To use this tool you must first construct the two hyperbolas. Selecting the intersection tool you must then point to the two hyperbolas. Once you point to the second the intersection points automatically pop up:



*Remark:* The construction works by computing the equation for the two hyperbolas with respect to the coordinate system. This is done by randomly selecting three points on each hyperbola. The tool does not check if they actually are hyperbolas. So the tool will actually attempt to compute 'intersection points' between any loci, but of course in that case the points computed will be the intersection points between two hyperbolas passing through triples of random points on the loci, so they will be useless. The tool does not invoke a general algorithm for construction of intersection points between loci but is tailor-made for Minkowski hyperbolas, which have simple quadratic equations. As a consequence the intersection point can be computed using the formula for solving a quadratic equation.

## Intersecting a straight line and a hyperbola: Linept1 Linept2 Hyp

This is the basic tool for constructing an intersecting point between a straight object such as a line, a ray or a segment and a Minkowski hyperbola. The position of the intersection points are computed using an underlying coordinate system, so when this macro is invoked a coordinate system automatically pops up. You can hide the coordinate system if it disturbs your investigations. To use this tool you must first construct the hyperbola. Selecting the intersection tool you must then point to the two line points specifying the direction of the straight object. Once you then point to the hyperbola the intersection points automatically pop up:

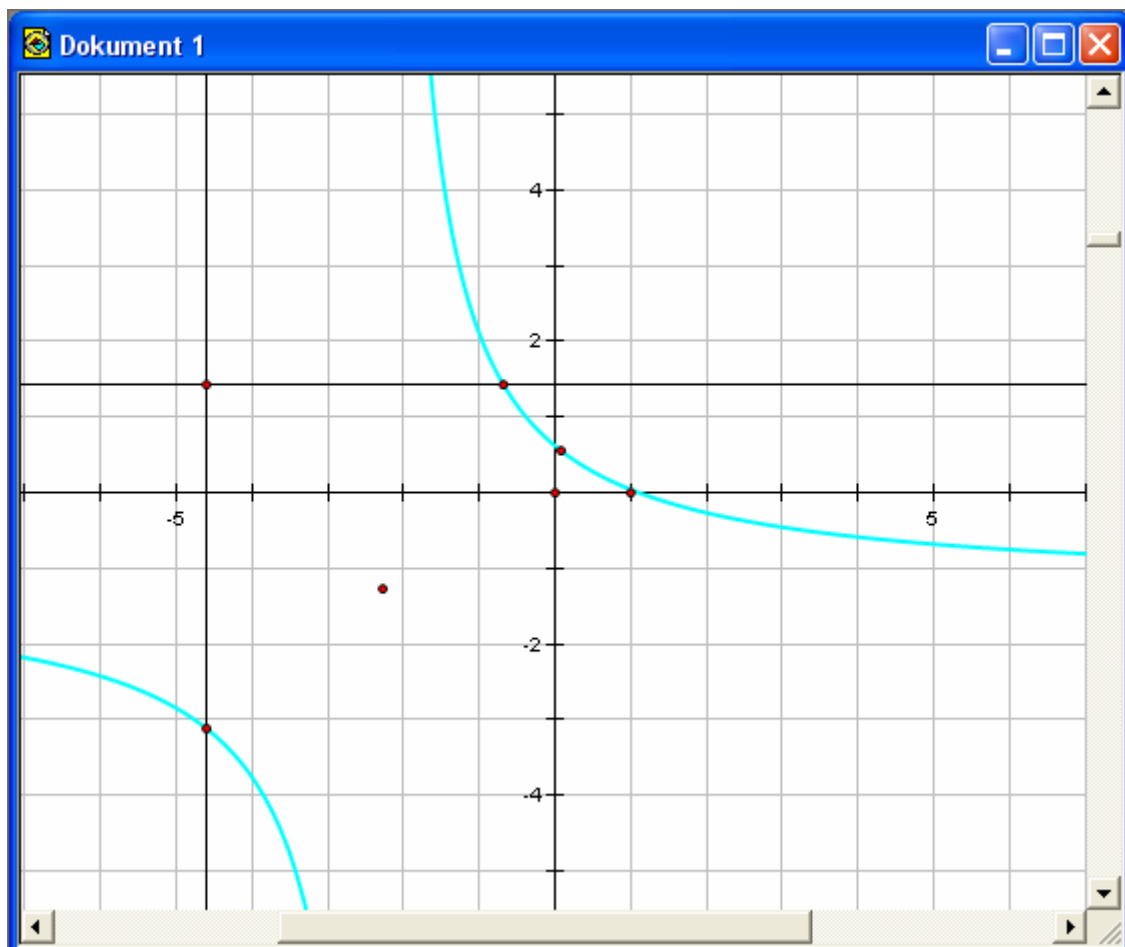


*Remark:* The construction works by computing the equation for the hyperbola with respect to the coordinate system. This is done by randomly selecting three points on the hyperbola. The tool does not check if it actually is a hyperbola. So the tool will actually attempt to compute 'intersection points' between a straight object and any locus, but of course in that case the points computed will be the intersection points between the straight object and the hyperbola passing through a triple of random points on the locus, so they will be useless. The tool does not invoke a general algorithm for construction of intersection points between straight lines and loci but is tailor-made for Minkowski hyperbolas, which have simple quadratic equations. As a consequence the intersection point can be computed using the formula for solving a quadratic equation.



## Intersecting a null line and a hyperbola: Basept Hyp

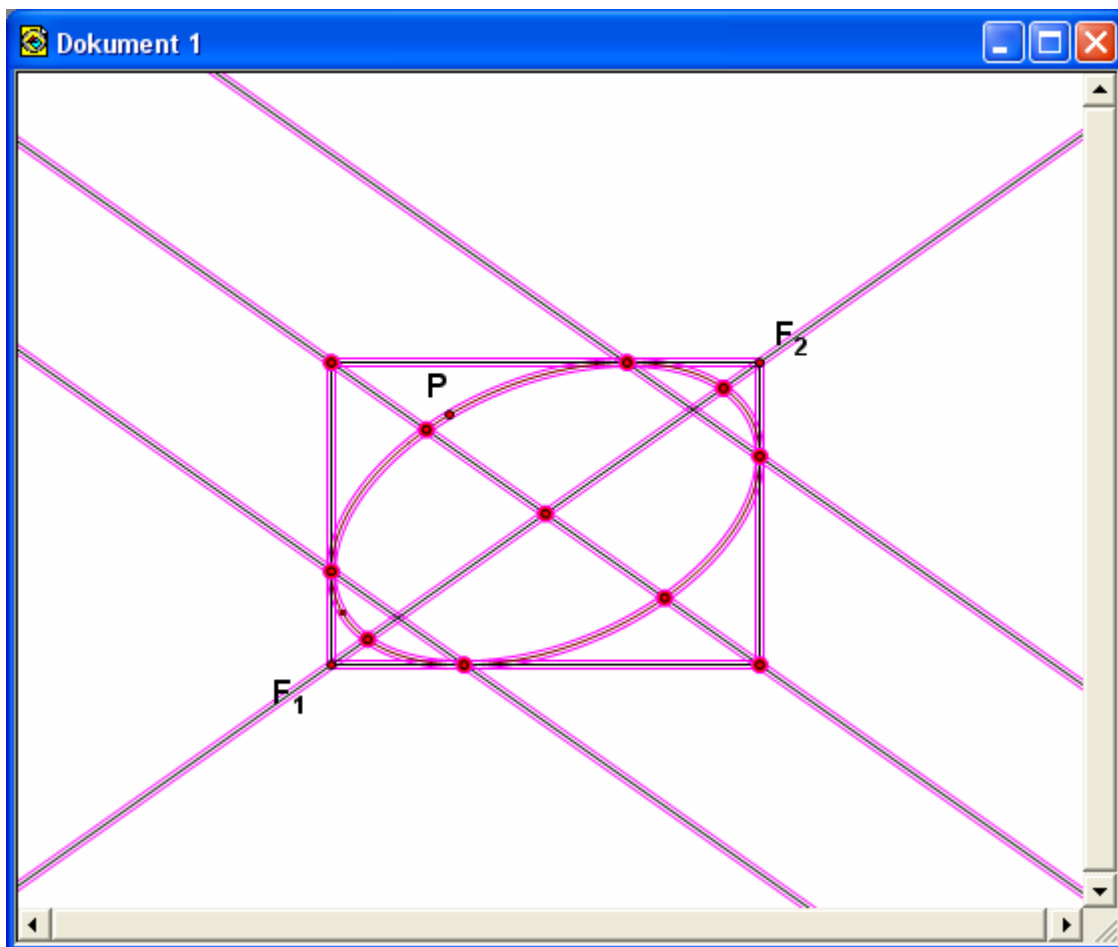
This is the basic tool for constructing an intersecting point between a null line, i.e. a horizontal or vertical line and a Minkowski hyperbola. The position of the intersection points are computed using an underlying coordinate system, so when this macro is invoked a coordinate system automatically pops up. You can hide the coordinate system if it disturbs your investigations. To use this tool you must first construct the hyperbola. Selecting the intersection tool you must then point to a base point through which the null line passes. The null lines pop up automatically while selecting the position of the base point. Once you then point to the hyperbola the intersection points for both the horizontal and vertical null line automatically pop up:



*Remark:* The construction works by computing the equation for the hyperbola with respect to the coordinate system. This is done by randomly selecting three points on the hyperbola. The tool does not check if it actually is a hyperbola. So the tool will actually attempt to compute 'intersection points' between a null line and any locus, but of course in that case the points computed will be the intersection points between the null line and the hyperbola passing through a triple of random points on the locus, so they will be useless. The tool does not invoke a general algorithm for construction of intersection points between null lines and loci but is tailor-made for Minkowski hyperbolas, which have simple equations, where both  $x$  and  $y$  can be trivially isolated.

## Minkowski Ellipse: F1 F2 Pt

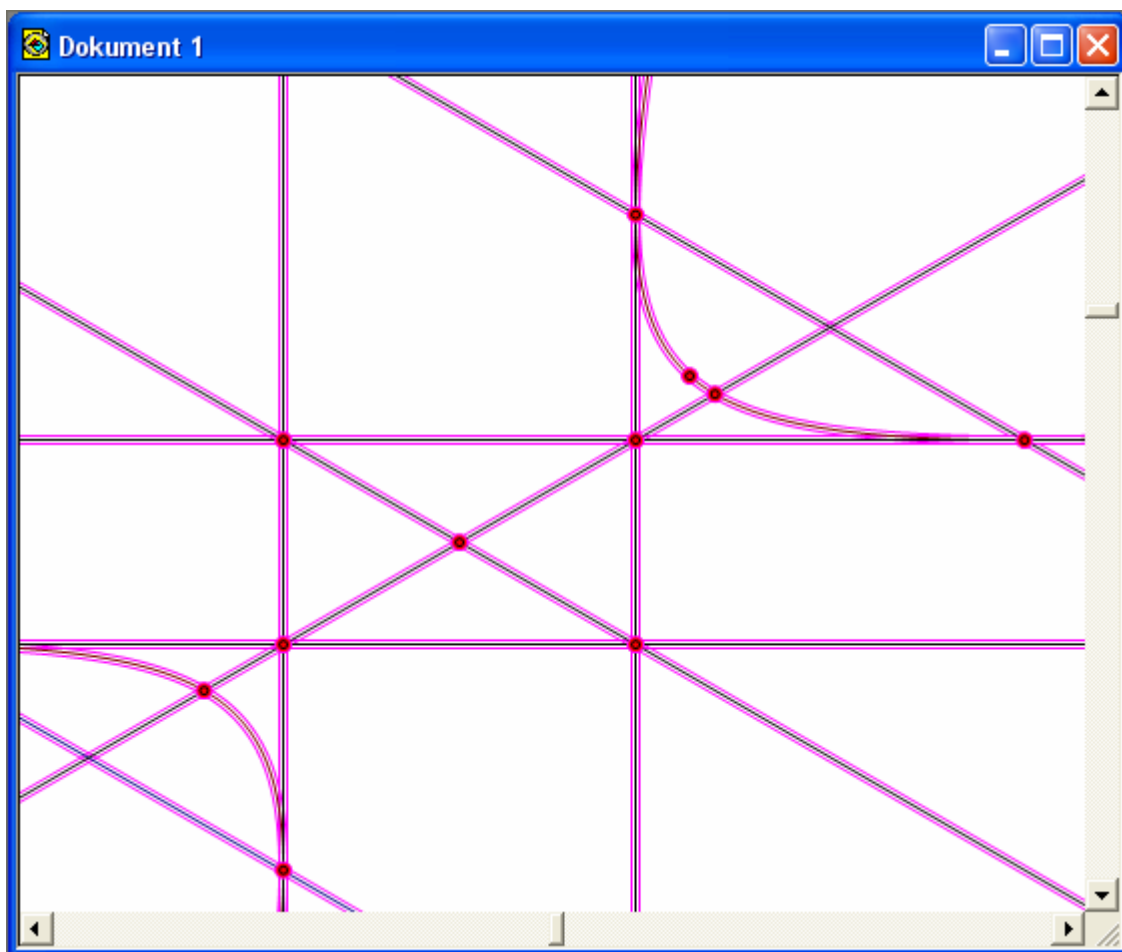
This is a supplementary tool which allows the construction of an ellipse from a pair of focal points and a boundary point. To use this tool you must therefore select three points, the two foci and the boundary point. Once the first focal point has been selected the central axes and the enclosing rectangle pop up. Once the second focal point has been selected the ellipse pops up together with some auxiliary points and lines including vertices and axes. Notice however that not all positions are admissible: To create an ellipse, the boundary point must be within the enclosed rectangle consisting of the null lines passing through the foci. So if nothing happens try to move the boundary point to a more suitable position.



Conic sections in Minkowski Geometry have a somewhat different behavior from conic section in Euclidean Geometry (although as affine curves they are of course identical!) The most important difference is the existence of null lines and null points, i.e. points on the conic section where the tangent are horizontal or vertical. These null lines intersect each other at the foci (so e.g. the ellipse has in fact four foci!). Similarly the lines passing through the adjacent null points form the directrices. As you can see from the illustration the position of the foci are also reversed from Euclidean geometry: For an ellipse the Minkowski foci fall outside the ellipse. As a consequence the eccentricity is now larger than 1 (with the Euclidean circle having eccentricity  $\sqrt{2}$ ).

## Minkowski Hyperbola: F1 F2 Pt

This is a supplementary tool which allows the construction of a hyperbola from a pair of focal points and a boundary point. To use this tool you must therefore select three points, the two foci and the boundary point. Once the first focal point has been selected the central axes and the enclosing rectangle pop up. Once the second focal point has been selected the hyperbola pops up together with some auxiliary points and lines including vertices and axes. Notice however that not all positions are admissible: To create a hyperbola, the boundary point must be within the diagonal region associated with the second focus (outside the enclosed rectangle consisting of the null lines passing through the foci). So if nothing happens try to move the boundary point to a more suitable position.



Conic sections in Minkowski Geometry have a somewhat different behavior from conic section in Euclidean Geometry (although as affine curves they are of course identical!) The most important difference is the existence of null lines and null points, i.e. points on the conic section where the tangent are horizontal or vertical. These null lines intersect each other at the foci. Similarly the lines passing through the adjacent null points form the directrices. As you can see from the illustration the position of the foci are also reversed from Euclidean geometry: For a hyperbola the Minkowski foci fall in the same region as the centre point. As a consequence the eccentricity is now smaller than 1 (with the equiangular hyperbola having eccentricity 0).